The Rules

\[
\begin{align*}
< \text{skip}, \sigma > & \rightarrow < E, \sigma > \\
< u := t, \sigma > & \rightarrow < E, \sigma[u := \sigma(t)] > \\
< S_1, \sigma > & \rightarrow < S_2, \tau > \\
< S_1; S, \sigma > & \rightarrow < S_2; S, \tau > \\
E; S & \equiv S \\
< \text{if } B \text{ then } S_1 \text{ else } S_2 \text{ fi}, \sigma > & \rightarrow < S_1, \sigma > \text{ where } \sigma \models B \\
< \text{if } B \text{ then } S_1 \text{ else } S_2 \text{ fi}, \sigma > & \rightarrow < S_2, \sigma > \text{ where } \sigma \models \neg B \\
< \text{while } B \text{ do } S_1 \text{ od}, \sigma > & \rightarrow < S_1; \text{while } B \text{ do } S_1 \text{ od}, \sigma > \text{ where } \sigma \models B \\
< \text{while } B \text{ do } S_1 \text{ od}, \sigma > & \rightarrow < E, \sigma > \text{ where } \sigma \models \neg B
\end{align*}
\]

Reductions

Reduce the following programs according to the semantic rules given.

Problem 1)
\[
< \text{if } x > y \text{ then } m := x \text{ else skip fi; if } x < y \text{ then } m := y \text{ else skip fi;}, \{x:=10, y:=30\}>
\]

Problem 2)
\[
<n := 0; \text{while } x > 1 \text{ do } x := x/2; n := n+1 \text{ od, } \{x:=8\}>
\]

Problem 3)
(Don’t spend too much time on this one.)
\[
<p := 1; n := 3; \text{while } n > 1 \text{ do } p := p \times x \text{ od, } \{x:=3\}>
\]
Make your own rules!

Problem 4) Write a rule to explain the when B S statement. It executes S only if B is true.

Problem 5) Write a rule for do S while B od. It is like while, but executes S at least one time.

Church Rosser

Problem 6) Consider this semantic rule:

\[ x_1 \circ x_2 \circ \cdots \circ x_i \circ x_{i+1} \circ \cdots \circ x_n \rightarrow x_1 \circ x_2 \circ \cdots \circ (x_i \ast x_{i+1}) \circ \cdots x_n \]

Does it have the Church-Rosser property? Try to prove it.

Problem 7) Consider this semantic rule:

\[ x_1 \circ x_2 \circ \cdots \circ x_i \circ x_{i+1} \circ \cdots x_n \rightarrow x_1 \circ x_2 \circ \cdots \circ (x_i - x_{i+1}) \circ \cdots x_n \]

Does it have the Church-Rosser property? Try to prove it.