Part 1 --- Ambiguous Grammars

These grammars are ambiguous. Prove that they are ambiguous by giving an input with two parse trees. Give an equivalent grammar that is unambiguous.

Example 1)

\[ E \rightarrow E + E \]  
\[ \quad | \quad E \times E \]  
\[ \quad | \quad - E \]  
\[ \quad | \quad i \]

This grammar is ambiguous. Consider the input 2 + 5 * 3. (Let i be an integer.) There are two possible trees for this:

\[
\begin{align*}
E & \rightarrow E + E \\
E & \rightarrow E \times E \\
2 & \rightarrow E + E \\
5 & \rightarrow E \times E
\end{align*}
\]

Tree 1  
Tree 2

Rewrite the grammar so that * and + have their usual precedences, and associate to the left. Unary minus binds most tightly.

\[ E \rightarrow E + T \]  
\[ \quad | \quad T \]  
\[ T \rightarrow T + F \]  
\[ \quad | \quad F \]  
\[ F \rightarrow - F \]  
\[ \quad | \quad i \]

Problem 1)

\[ E \rightarrow E a E \]  
\[ \quad | \quad E b E \]  
\[ \quad | \quad E x E \]  
\[ \quad | \quad E y E \]  
\[ \quad | \quad i \]

The grammar is ambiguous. Here are two trees that demonstrate an ambiguity, for input 2 a 5 x 3.

\[
\begin{align*}
E & \rightarrow E a E \\
E & \rightarrow E x E \\
2 & \rightarrow E x E \\
5 & \rightarrow E x E
\end{align*}
\]

Tree 1  
Tree 2

Rewrite the grammar so that a has highest precedence, associating to the left. Next is b, associating to the left. Then we have y associating to the right. Let x have the lowest precedence, and associate to the right.
\[ E \rightarrow F x E \]
\[ F \rightarrow G y F \]
\[ G \rightarrow G b H \]
\[ H \rightarrow H a i \]

Part 2 --- First and Follow Sets

Calculate the first and follow sets for these grammars.

**Example 2)**
\[
S \rightarrow a E b \\
| \ x \\
E \rightarrow x y \\
| \ \epsilon
\]

<table>
<thead>
<tr>
<th>Symbol</th>
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<tr>
<td>S</td>
<td>a, x</td>
<td>$</td>
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<tr>
<td>E</td>
<td>x, \epsilon</td>
<td>b</td>
</tr>
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**Example 3)**
\[
S \rightarrow a E F b \\
| \ x E F \\
E \rightarrow x y \\
| \ \epsilon \\
F \rightarrow E z q \\
| \ w S
\]

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<tr>
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**Problem 2)**
\[
S \rightarrow a F E b \\
| \ x F E \\
E \rightarrow x y \\
| \ \epsilon \\
F \rightarrow E z q \\
| \ w S
\]

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