Part 1 --- Ambiguous Grammars

Consider the following ambiguous grammar.

**Example 1)**

\[
E \rightarrow E + E \\
| E \ast E \\
| - E \\
| i
\]

**Problem 1)** Prove that this grammar is ambiguous by giving an input with two parse trees.

**Problem 2)** The instructor will give you a version of this grammar that is not ambiguous. We will let \( \ast \) and \(+\) have their usual precedences, and associate to the left. Unary minus binds most tightly. Show that your input has only one parse tree now.

\[
E \rightarrow E + F | F \\
F \rightarrow F \ast U | U \\
U \rightarrow - U | i
\]

**Problem 3)** Stratification removed the ambiguities. How did it do that?
Here is a new grammar for you.

\[ E \rightarrow E \ a \ E \quad | \quad E \ b \ E \quad | \quad E \ x \ E \quad | \quad E \ y \ E \quad | \quad i \]

**Problem 4)** Prove that this grammar is ambiguous by giving an input that has two parse trees.

**Problem 5)** Stratify the grammar to remove ambiguity. Let \( a \) have highest precedence, associating to the left. Next is \( b \), associating to the left. Then we have \( y \) associating to the right. Let \( x \) have the lowest precedence, and associate to the right. Show that your sentence has only one parse tree.
Part 2 --- First and Follow Sets

Calculate the first and follow sets for these grammars.

Example 2)

\[
S \rightarrow a \ E \ b \\
| \ x \\
E \rightarrow x \ y \\
| \ \epsilon
\]

Problem 6)

\[
S \rightarrow a \ E \ F \ b \\
| \ x E \ F \\
E \rightarrow x \ y \\
| \ \epsilon \\
F \rightarrow E \ z \ q \\
| \ w \ S
\]

Problem 7)

\[
S \rightarrow a \ F \ E \ b \\
| \ x F \ E \\
E \rightarrow x \ y \\
| \ \epsilon \\
F \rightarrow E \ z \ q \\
| \ w \ S
\]